

# **CCS-3 Modeling, Algorithms, and Informatics**

- Computer Science and Mathematics Research
- Theoretical and Applied
  - High Performance Computing, Performance, Architecture
  - Machine Learning
  - Knowledge Systems and Computational Linguistics
  - Computational Biology and Bioinformatics
  - Multilevel Solvers
  - Quantum Information Processing







#### **Tidbits**

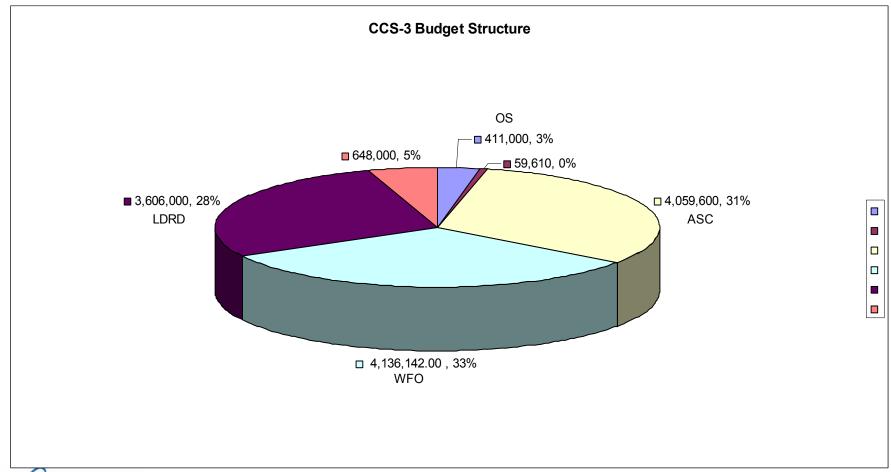
- 33 staff members, O(10) students, 4 postdocs
- Programmatic work in NW and TR
- Emphasis on scholarly work: in the year between DRC 2005 and DRC 2006, published 34 peer-reviewed journal papers, 20 peer reviewed conference proceedings papers, 3 edited books.
- Emphasis on software development in many areas
- Connected with various relevant scientific communities directly and through a myriad of internal and external collaborations
- O(50) charge codes...







# **CCS-3 Budget**









# Modeling, Algorithms, & Informatics (CCS-3)

Adolfy Hoisie, Group Leader Francis J. Alexander, Deputy Group Leader Erika Maestas, Staff Assistant

Margaret Tyler, Administrative Specialist Justin Chavez, UGS Administrator Hugh Greenberg, System Administrator

#### **Performance & Architecture** Lab (PAL)

Adolfy Hoisie, TL

Kevin Barker Eitan Frachtenberg Mike Lang Song Jiang Greg Johnson

Darren Kerbyson Scott Pakin Jose Sancho

#### **Machine Learning & Pattern** Recognition

Patrick M. Kelly, TL

Olaf Lubeck Pat Fasel Rich Fortson Reid Rivenburgh John Hogden Clint Scovel James Howse Ingo Steinwart Don Hush

#### **Quantum & Classical Information Sciences**

Marion Kei Davis, TL

**Howard Barnum** Chris Brislwan **Leonid Gurvits** Paul Pedersen

#### **Solvers**

Michael Pernice, TL

**Bobby Philip** 

#### **Knowledge & Information Systems Science**

Cliff A. Joslyn, TL

Karin Verspoor Sue Mniszewski Shou-De Lin Ari Rosenberg

#### **Information Physics** & Modeling

Francis J. Alexander, TL

Marian Anghel John Middleditch Allon Percus

#### **Computational Biology** & Bioinformatics

Michael Wall, TL

**Dengming Ming** Ilya Nemenman Nikolai Sinitsyn









#### **Performance and Architecture Lab (PAL)**

#### Research Areas:

- High performance computing
- Performance Analysis
- Performance Modeling
- System architecture
- Interconnect networks
- Scheduling
- System software

#### Recent Projects:

- Performance of leading supercomputers, advanced architectures, IB, optical networks, system software performance, tool development for performance, OS scheduling for multi-cores, quantifying and exploiting communication-computation overlap, microprocessor architecture
- ASC, DARPA, Office of Science









# Performance and Architecture Lab: Recent Accomplishments

- Systems analyzed (measured and modeled):
  - Blue Gene/L (LLNL), RedStorm (SNL), Purple (LLNL)
  - ASC L2 Milestone on Blue Gene/L and RedStorm successfully completed
  - Multi-core analysis of AMD and Intel
- Workload modeled increased
- Novel Networks explored:
  - Optical Circuit Switched network (OCS) with IBM, SC05
- Novel systems analyzed:
  - IBM PERCS, HPCS, multitude of configurations explored, used in HPCS review and Phase 3 proposal
  - Accelerators case study of S<sub>N</sub> transport on Clear-speed CSX600 completed
- World-class publication record in 2005/6:
  - 14 Journal Papers, 20 Conference Papers, and 2 edited books,
  - includes 3 in IEEE Transactions (Computers and Parallel & Distributed Processing)
- Installation of 1024 core AMD system with Infiniband
  - For performance analysis, development, and software stack analysis





#### **Quantum and Classical Information Sciences**

#### Research Areas:

- Quantum Information Processing
- Theory of Multirate Digital Filter Banks and Wavelet Transforms

## **Example Applications:**

- Quantum and Classical Cryptography Quantum Physics
- Quantum and Classical Complexity Digital Image Source Coding
- Quantum ComputingRF Remote Sensing







#### **Recent Contributions**

- Settled long-standing conjecture on NP-hardness of checking whether a given bipartite mixed quantum state is entangled
  - Implications for QIP, signal processing, and computer vision
- Quantification of robustness of quantum entanglement
  - Implications for both theory, and practical implementation of quantum cryptography and computing
- Found deep connections between fundamental classical complexity problems and quantum entanglement.
  - May lead to tractable, deterministic classical algorithms for a large class of problems solvable by quantum computing
- Proposal for experimental continuous-variables quantum key distribution
  - May greatly increase secret key rate by using telecom-industry technology rather than single-photon states and detectors







## **Knowledge and Information Systems Science**

- Knowledge discovery and extraction:
  - Emphasis on semantic information
  - Hybrid approach combining statistical, numerical, and quantitative with symbolic, logical, and qualitative techniques
- Ontologies and Conceptual Semantic Systems: Discrete mathematical, combinatorial, and order theoretical approaches
- Computational Linguistics: Text extraction, knowledge discovery, search and retrieval, corpus management
- Database Analysis: User-guided knowledge discovery in complex, multi-dimensional data spaces
- Software Architectures: Parallel and high performance algorithms for knowledge systems, simulations



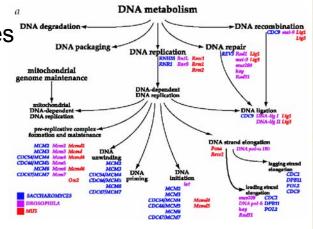


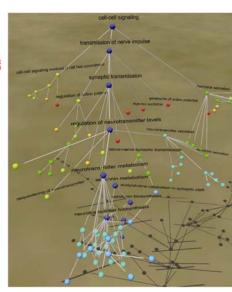


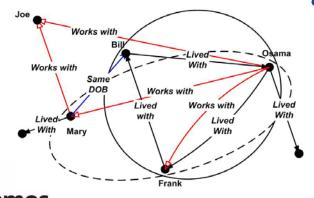
# **Ontologies and Knowledge Representation**

#### Bio-Ontologies

- Discrete math approaches to large semantic hierarchies
- Applied lattice theory
- Information visualization (D-4 collaboration)
- For protein function inference







#### Semantic Network Databases

- Statistical techniques for typed path analysis
- Abnormal node and connection discovery
- Intelligence analysis and homeland security





# **Computational Linguistics and Text Extraction**

#### Los Alamos Semantic Event Recognizer (LASER)

- Uses templates for relation extraction from text
- Unsupervised and weakly supervised machine learning techniques for template creation

In 3T3-L1 adipocytes, a low level of serine phosphorylation of stat3 on residue 727 was observed and was markedly enhanced by ins or os.

ins phosphorylate stat3
os phosphorylate stat3

#### Extraction of Motivation and Intent from Text

- Statistical content analysis of phrases annotated by affect, mood, or emotion
- In interaction with syntactic/phrasal structure

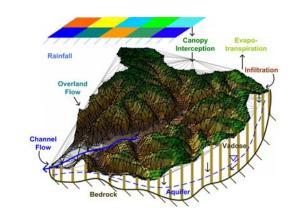
[The Party]<sub>subject</sub> [denounces]<sub>verb</sub> [[the terrorizing of innocent people] and [the use of emergency law [against honest citizens]]]<sub>object</sub>.

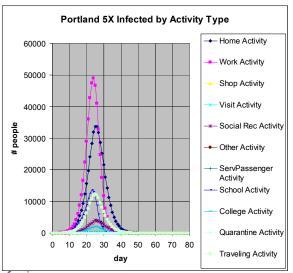




## Parallel and High Performance Scientific Algorithms

- High Resolution Physically-Based Model of Semi-Arid River Basin Hydrology
  - Parallelization of watershed models for used for validation and prediction
  - Institutional Computing resources, TLC, Lambda, and Mauve





- Epidemic Simulation System (EpiSimS)
  - Public Health Module of NISAC Urban Infrastructure Suite (UIS)
  - Simulation of metropolitan disease spread reflecting social contacts and disease transmission







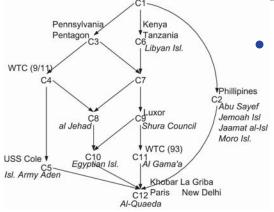
# **Cubic Lattice Data Analysis**

#### Database hypercubes

- User-guided knowledge discovery in high-dimensional databases
- OnLine Analytical Processing (OLAP)
- Cubic lattice formulation

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	Grand		203,284.22	Drink 4,764.44	42,586.16	Non-Consumable 10,943.70	Drink	Food 34,876.63		WA Drink 8,362.00	Food 69,502
	Grand 1	Item:1	203,284.22 18,218.77	Drink 4,764.44 746.35		Non-Consumable	Drink 4,385.66	34,876.63	9,696.88	WA Drink	Food 69,502.
		Item:1 20319	203,284.22 18,218.77 4,042.96	Drink 4,764.44 746.35	42,586.16	Non-Consumable 10,943.70	Drink	34,876.63	9,696.88	WA Drink <b>8,362.00</b> 471.67	Food 69,502. 4,645
		Item:1 20319 21215	203,284.22 18,218.77 4,042.96 11,517.13	Drink 4,764.44 746.35	<b>42,586.16</b> 8,905.01	Non-Consumable 10,943.70 2,199.30	Drink 4,385.66	34,876.63	9,696.88	WA Drink 8,362.00	Food 69,502. 4,645
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		Item:1 20319 21215 22478 23598	203,284.22 18,218.77 4,042.96 11,517.13 988.92 11,634.92	Drink: 4,764.44 746.35 105.70 850.18	42,586.16 8,905.01 662.18 8,725.71	Non-Consumable 10,943.70 2,199.30 220.94 2,059.03	Drink 4,385.66	34,876.63	9,696.88	WA Drink <b>8,362.00</b> 471.67	Food 69,502 4,645
	1997	Item:1 20319 21215 22470 23598 23680	203,284.22 18,218.77 4,042.96 11,517.13 988.82 11,634.92 5,132.90	Drink: 4,764.44 746.35	42,586.16 8,905.01 662.18	Non-Consumable 10,943.70 2,199.30 220.94 2,059.03	Drink 4,385.66 324.72	34,876.63 2,696.76	9,696.88 1,021.48	WA Drink <b>8,362.00</b> 471.67	Food 69,502. 4,645
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å Time - 1	1997	Item:1 20319 21215 22470 23598 23608 27694 28206 30268	203,284.22 18,218.77 4,042.96 11,517.13 968.82 11,634.92 5,132.90 16,151.04 1,049.46 5,781.96	Drink: 4,764.44 746.35 105.70 850.18	42,586.16 8,905.01 662.18 8,725.71	Non-Consumable 10,943.70 2,199.30 220.94 2,059.03	Drink 4,385.66 324.72	34,876.63 2,696.76	9,696.88 1,021.48	WA Drink 8,362,00 471.67 990.57	Food 69,502. 4,645 8,596 696 3,901
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# Formal Concept Analysis

- Extracting hierarchical structure of relational data
- Ontology induction







## **Computational Biology and Bioinformatics**

- Capabilities
  - Mathematical Analysis and Modeling
  - Statistical Physics
  - Complex Systems and Nonlinear Dynamics
  - Information and Learning Theory
- Applications
  - Genetic Regulatory Networks
  - Biomolecular Interactions
  - Protein Structure and Function
  - Neuroscience
- Projects
  - LDRD-DR (Wall, PI) "Computational Methods for Protein Function Inference"
  - LDRD-ER (Wall, PI) "Design Principles of Genetic Regulatory Networks"





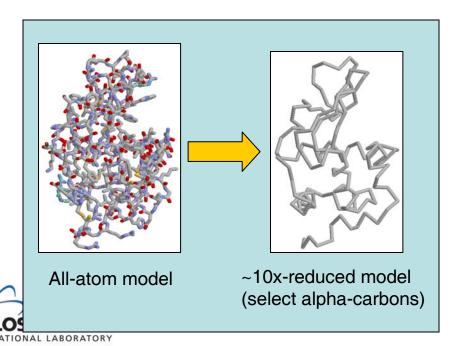


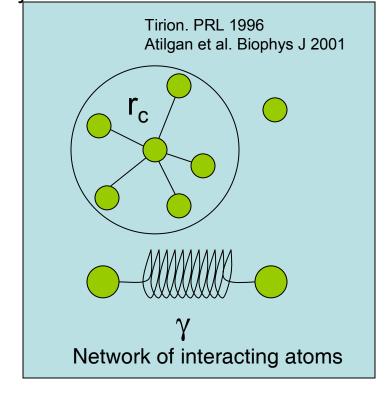
# **Coarse-Grained Modeling of Proteins**

- Extract key properties of detailed models for use in large-scale simulations
  - Molecular signaling mechanisms
  - Free-energy estimation

Make simulation of large macromolecular systems feasible

Example: Elastic Network Model



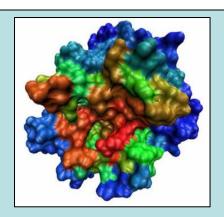


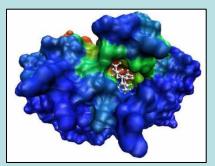




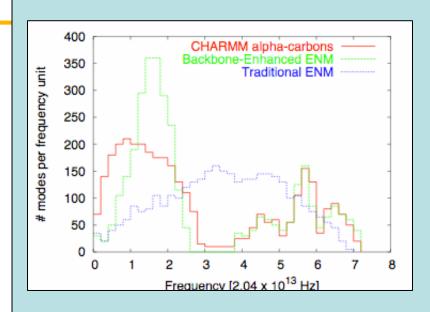
#### **Improved Coarse-Grained Model**

- Discovered a problem with the elastic network model
  - Incorrect density-of-states
- Fixed the problem
  - Strengthen backbone interactions
- Applied the new model to analyze communication between binding sites in proteins





Binding at one site (left) causes a large change in the second site (right)



Unimodal density of states (blue) disagrees with all-atom model (red)

Bimodal distribution is recovered by strengthening interactions between backbone neighbors (green)

D. Ming & M.E. Wall. Phys Rev Lett 95 (2005) 198301







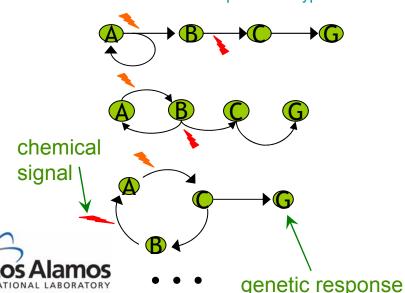


## Information processing in biochemical circuits

- How many bits can be transmitted by a realistic stochastic biochemical network?
   How many different cell fates can be encoded?
  - With constrained molecular copy number?
  - With constrained time delays?
- How does this number depend on the network topology? (Are there special topologies?)
- Is this number robust to parameter fluctuations?

#### Analyze all topologies

exactly one promoter per gene, each TF binds to one promoter type



Analyze steady states g=g(c) for all signals

$$\frac{dg}{dt} = -R_g g + a_0 + \alpha (\{g, c\}) = 0$$

Add Langevin noise and evaluate

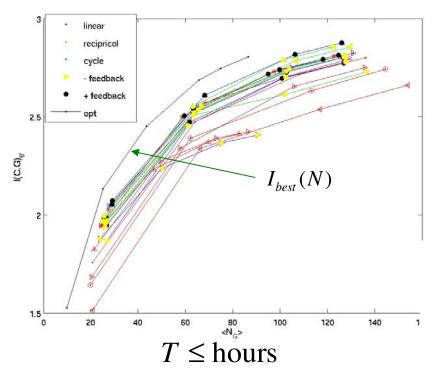
$$\widehat{\theta} = \arg\max_{\theta} I(C; G \mid \theta) - \lambda_1 N - \lambda_2 T$$

$$I_{\text{max}} = I(C; G \mid \theta)$$





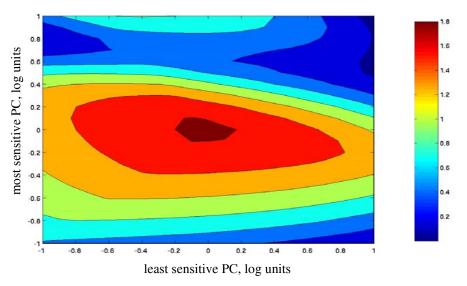
# All networks achieve (almost) maximum MI



All topologies achieve mutual info close to the maximum possible (e.g., at *N*=80 the max is 2.7 bits, while *the worst* topology allows about 2.4 bits.

#### Is MI sensitive to parameters?

- Almost 10-fold parameter changes may still lead to  $I \ge 0.8I_{best}$
- High *I* is generic! No fine-tuning.









#### **Important Research Directions**

- Computational methods for protein function inference
- Reverse engineering of biological networks
- Information processing in biological networks
- Mathematical modeling of gene regulation
- Coarse-grained modeling of biological systems
- Free-energy estimation for protein interactions
- Neural coding and animal learning
- Intelligent computing







#### **CCS-3 Solvers Team**

#### Capabilities:

- Full spectrum of linear/nonlinear solver methods
  - Multigrid, Krylov subspace methods, domain decomposition, Newton-Krylov methods
  - Special focus on optimal multilevel solvers for AMR grids
    - FAC, AFAC, AFACx methods as preconditioners
- Use with implicit time integration
- Adaptive mesh refinement
  - Grid management frameworks (SAMRAI)
  - Feature-based refinement criteria
  - Verified second-order spatial discretization schemes
- Ancillary software products
  - SAMRSolvers, SAMRUtilities
  - Solver software interoperability (PETSc, KINSOL)





# **CCS-3 Solvers Team Projects**

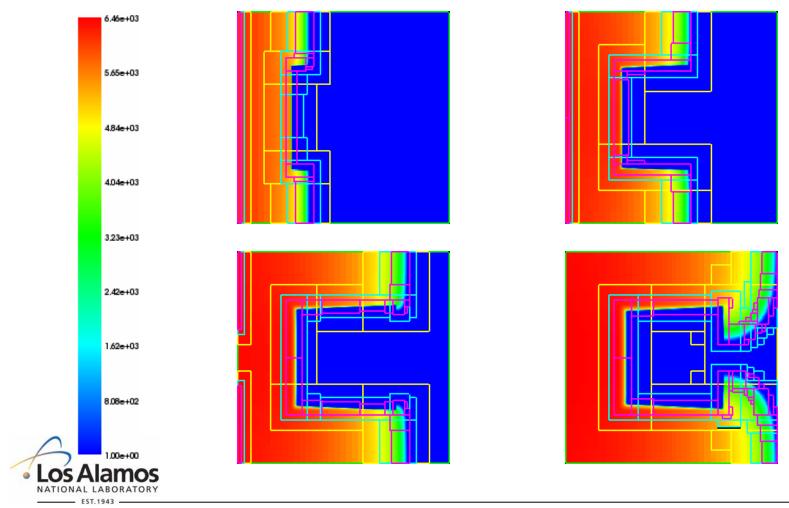
- Application of implicit AMR to Marshak wave problem
  - Produces same solution with 80-90% fewer mesh points
  - AMR up to 10x faster than equivalent fine grid calculation
- Application of implicit AMR to reduced resistive MHD
  - Goal: enable study of magnetic reconnection for low resistivities at unprecedented spatial resolution
  - First-of-a-kind simulation of tearing mode case
    - Showed weak dependence of iteration counts on mesh configuration
  - Supported by LDRD
- Comparison of FAC, AFAC with AMG on AMR grids
  - Use Marshak wave example
  - Try to quantify potential impact of future programmatic investment in geometric methods
  - Supported by WSR







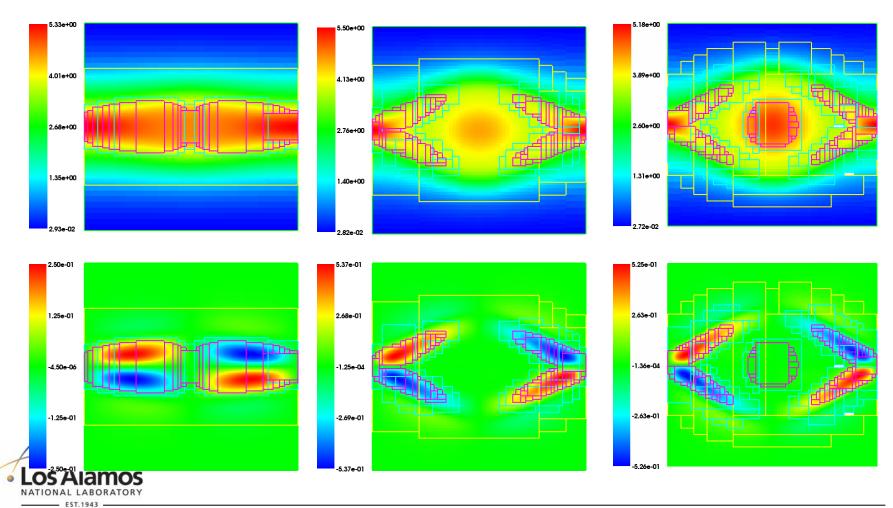
# Marshak Wave Sample Calculation







# **Tearing Mode Sample Calculation**







## **Machine Learning Team**

#### Research Areas:

- Computational Learning Theory
- Data Driven Modeling
- Statistical Inference
- Pattern Recognition
- Anomaly Detection

- Neural Networks
- Digital Image/Signal Processing
- Image Understanding
- Clustering Algorithms
- Data Analysis

## **Example Applications:**

- Satellite Image Analysis / Automated Tracking of Urban Growth Trends
- Fraud Detection (Tax Returns, Medicare Submissions)
- OCR Improvement for Degraded Document Images
- Automated Language Identification from Document Images
- Speech Recognition

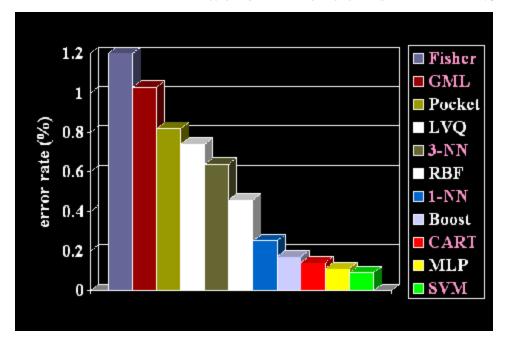




## **Basic Research in ML (Recent Contributions)**

- First Complete Solution Method for the generalized version of the supervised classification problem
  - Near-Optimal Performance is Guaranteed
  - Run-Time Guarantees that are both Feasible & Efficient

#### DARPA Intrusion Detection Data Set



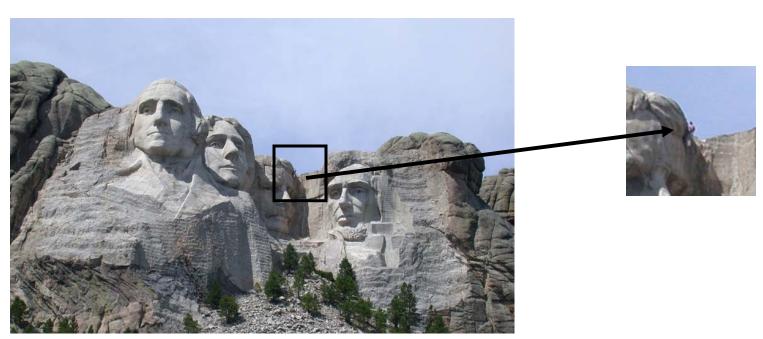






# **Basic Research in ML (Recent Contributions)**

 First Practical Anomaly Detection Framework that provides a method for validating anomaly detector performance









# **Current Image Analysis Efforts**

- Image Comparison Methodologies (FEEMADS)
  - Simulated Data vs. Experimental Results
  - Contributed validation techniques for three separate Level 2 ASC milestones
- Image Analysis / Pattern Recognition
  - "Find me all of the other things in the image that look like this..."







# **Information Physics and Modeling Team**

#### Research Areas:

- Variational Modeling of Large-Scale Nonlinear Systems
- Multiscale / Hybrid Numerical Methods
- Statistical Physics of Algorithms
- Statistical Physics (Foundations)

- Nonlinear Prediction
- Computational Kinetic Theory/Fluid Dynamics
- Accelerated Monte Carlo Methods
- Optimal Estimation / Inverse problems

## Applications:

- Non-convex optimization
- Phase Transition Kinetics and Dynamics
- Earthquake / Fault Modeling
- Powergrid Modeling and Analysis
- Source Identification /History Reconstruction
- Fast Langevin Molecular Dynamics
- Pulsar Astrophysics "Starquakes"



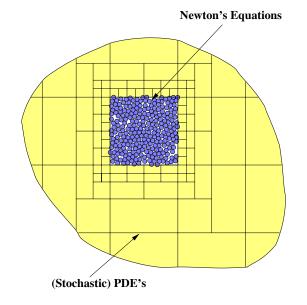


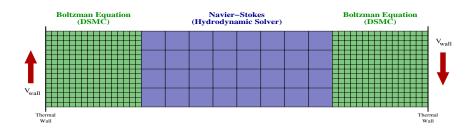


## **Hybrid Methods**

The accurate and efficient simulation of multiscale phenomena\*

- Different spatial regions require different physics/resolutions
- Use computationally cheapest, <u>valid</u>, method in each region
- Dynamically couple methods
- Computational Gain =V<sub>macro</sub>/V<sub>micro</sub>







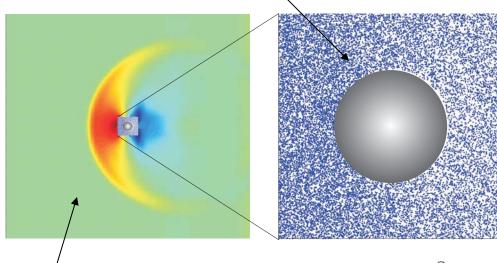




# **Hybrid Methods**

# **Boltzmann Equation**

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_r f + \frac{\mathbf{F}}{m} \nabla_v f = \Omega_{coll}$$



# **Euler / Navier-Stokes**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{\Pi} = \mathbf{0}$$



$$\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho \mathbf{e}\mathbf{u}) + \nabla \cdot \mathbf{q} + \mathbf{P} : \nabla \mathbf{u} = \mathbf{0}$$

